CALCULATING THE CALIBRATION CURVES OF HIGH-PRESSURE DEVICES WITH PROFILED ANVILS

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A determination of the calibration curves of high-pressure devices is presented on the basis of an approximate calculation of the stressed state of points in the median plane (with respect to height) of a thin layer of an ideal generalized plastic material, compressed between rigid profiled anvils. A specific relationship is derived for the angle between the greatest principal stress and the median plane as a function of height and it is assumed that slip, retardation, and stagnation zones occur in the contact region. Recommendations are made in order to determine the boundaries of the stagnation zone for the case of profiled anvils. The difference between the results of the calculation and experimental data is no greater than 8%.

In a number of papers [1, 2] devoted to the calculation of calibration curves for high-pressure devices of various kinds, the original stress equation was taken in a form based on the theory of plastic flow (yield) as applied to substances moving over rigid surfaces [3],

$$\partial p/\partial r = -2\tau_c/H. \tag{1}$$

where p and τ_c are, respectively, the pressure in the compressed material and the tangential frictional stress on the contact surface; H is the thickness of the compressed layer, being a smoothly (and very slightly) varying function of the coordinates.

On practically the whole of the contact surface (except for the central and boundary zones, the dimensions of which were of the order of the thickness of the compressed material) the frictional contact stresses were taken as equal to the shear yield stress of the compressed material, this stress being a function of pressure, i.e., $\tau_c = K(p)$. In order to allow for effects arising from the deformation of the plungers (pistons), the height of the compressed material was expressed in the form of a sum

$$H = h - 2\omega(r).$$

where $\omega(r)$ is the deformation of the plungers and h is the height of the compressed layer without taking account of deformation.

The pressure in the center of the compressed layer was calculated up to 55 kbar in [2], and the results were in excellent agreement with experiment. According to earlier data [1] for pressures of over 60 kbar the results of such calculations exceed the experimental values, and this difference increases with rising pressure. One of the reasons for the discrepancy may be that the actual influence of the deformation of the plungers on the flow of the compressed layer is more complicated than a simple increment in the height of the compressed layer, as implied by Eq. (1) [1, 2].

In the papers indicated in the foregoing discussion, apparatus with only a very thin compressed layer (< 1 mm) was the subject of the calculations. Very slight deformations of the plungers may have a considerable influence on the character of the flow in the compressed material; this influence was not adequately taken into account in the papers in question – hence the difference between the computed and experimental results. In

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other devices [4, 5] with profiled anvils and comparatively large transverse dimensions of the working space, the thickness of the compressed layer in the center of the apparatus is over 10 mm; the influence of the deformation of the plungers in these devices on the character of the flow in the compressed layer is much less severe than in devices with plane anvils.

For pressures between 0 and 100 kbar we may reasonably expect fair agreement between the experimental data and the results of the calculations for devices with profiled anvils, even without allowing for the effects of the deformation of the plungers; this greatly eases the computing procedure.

In the earlier papers [1, 2] the calculation was applied to devices with plane anvils, since in the original equation (1) the thickness of the compressed layer was a very slowly varying function of the coordinates. On profiling the ends of the plungers the thickness of the compressed material changes by a factor of 2-3 times; the character of the flow (and hence the pressure gradient) will then be considerably affected not only by the thickness of the compressed layer, but also by the angle between the outflow direction and the contact surface. Equation (1) is incapable of allowing for this effect.

As initial stress equation in this paper we shall use an equation derived from the differential equilibrium equations, and shall consider that the state of stress in the compressed material is nonuniform in the direction in which the plungers approach one another; the character of this nonuniformity is determined by the form of contact surface of the plungers.

The initial equation is derived on the assumption of axisymmetrical compression. The stressed state of an axisymmetrically compressed solid is described by differential equations of equilibrium expressed in cylindrical coordinates (r, θ, z), having the z axis along the direction of mutual approach of the plungers:

$$\begin{gathered}\partial\sigma_r/\partial r + \partial\tau_{rz}/\partial z + (\sigma_r - \sigma_\theta)/r = 0;\\ \partial\sigma_z/\partial z + \partial\tau_{rz}/\partial r + \tau_{rz}/r = 0,\end{gathered} \tag{2}$$

where σ_r, σ_θ, σ_z, τ_{rz} are, respectively, the radial, circumferential, axial, and tangential components of the stress tensor.

The directions of the components σ_r, σ_z, τ_{rz} lie in the axial plane rz, and on the basis of Mohr's theory may be expressed in the form of their well-known dependence on the average stress σ ($\sigma < 0$) in the plane of the axial section and the angle α made by the greatest principal stress in the axial plane with the positive direction of the r axis:

$$\sigma_r = \sigma + K\cos 2\alpha;\quad \sigma_z = \sigma - K\cos 2\alpha;\quad \tau_{rz} = K\sin 2\alpha, \tag{3}$$

where K is the shear yield stress of the compressed material and $\sigma = (\sigma_r + \sigma_z)/2$ is the average stress in the plane of the axial cross section (rz).

Accepting the condition of complete plasticity (Haar–Kármán) $\sigma_\theta = \sigma_1 = \sigma_2$ and a plastic mode of flow corresponding to the edge of the Tresca prism

$$\sigma_2 - \sigma_3 = 2K, \tag{4}$$

we obtain an expression for the circumferential stress

$$\sigma_\theta = \sigma + K. \tag{5}$$

The expressions (3) and (5) enable us to rewrite the system of equations (2) with four unknowns in the form of a system of two equations with two unknown functions σ and α :

$$\begin{gathered}\partial\sigma/\partial r = K\sin 2\alpha\cdot\partial 2\alpha/\partial r - K\cos 2\alpha\cdot\partial 2\alpha/\partial z + K(1-\cos 2\alpha)/r - \partial K/\partial r\cos 2\alpha - \partial K/\partial z\sin 2\alpha;\\ \partial\sigma/\partial z = \partial K/\partial z\cos 2\alpha - \partial K/\partial r\sin 2\alpha - K\sin 2\alpha\cdot\partial 2\alpha/\partial z - K\cos 2\alpha\cdot\partial 2\alpha/\partial r - K\sin 2\alpha/r.\end{gathered} \tag{6}$$

The analytical solution of both (2) and (6) in general form encounters serious difficulties of a mathematical nature; in solving applied problems it is therefore of no small importance to be in possession of approximate methods of solving the system (6).

In the majority of constructions of high-pressure apparatus, the compression is applied to materials having a median plane perpendicular to the direction in which the plungers approach one another. The system of equations (6) for points in the median plane becomes very much simpler on being expressed in a system of coordinates in which the rθ plane coincides with the median plane in question:

$$\partial(\sigma + K)/\partial r = -K\partial 2\alpha/\partial z; \quad \partial(\sigma - K)/\partial z = 0,$$

since in the median plane $\alpha = 0$, $\partial 2\alpha/\partial r = 0$.

The resultant system of equations is equivalent to the one equation

$$\partial(\sigma + K)/\partial r = -K\partial 2\alpha/\partial z$$

with two unknowns σ and α.

In order to find the unknown σ we must know the function α. From the boundary conditions we may determine the quantity Δ by which the angle 2α varies on varying z from 0 to H/2,

$$\Delta = -(\mathrm{arctg}\,(\tau_c/K_c) - 2\beta),$$

where $\beta = \arctan \partial/\partial r \cdot (H/2)$ is the angle made by the tangent to the profile of the anvil with the positive direction of the r axis and K_c is the shear yield stress of the compressed material at its contact with the plunger.

An unknown feature is the way in which the angle 2α varies from 0 to Δ as z varies from 0 to H/2. In this paper we shall express the mode of variation by means of a selected function $2\alpha = f(z)$. The form of the selected function $2\alpha = f(z)$ also determines the nonuniformity in the flow (yield) of the compressed material along the z coordinate. The selected function $2\alpha = f(z)$ should be such as to agree with existing solutions relating to the problem of the compression of plastic solids.

In solving the problem of the compression of a strip of constant height a linear dependence of the shear component on the coordinate z was obtained in [6], i.e.,

$$\tau_{rz} = K2z/H.$$

The linear dependence of the shear component on the z coordinate in the case of the compression of a strip of constant thickness corresponds to a relationship between the angle 2α and the z coordinate of the form (2z/H), i.e.,

$$2\alpha = \arcsin\,(2z/H). \tag{7}$$

Assuming that the angle 2α varies with z in accordance with the arcsin (2z/H) law for any inclinations of the contact surface, we may express the function $2\alpha = f(z)$ in the form

$$2\alpha = \Delta \frac{\arcsin\,(2z/H)}{\arcsin\,(2H/H2)} = \frac{\Delta}{\pi/2} \arcsin\,(2z/H).$$

Evaluating the derivative

$$\frac{\partial 2\alpha}{\partial z} = \frac{\Delta}{\pi/2} \frac{2/H}{\sqrt{1-(2z/H)^2}}$$

and substituting its value at z = 0 into Eq. (7), we obtain the original equilibrium equation in stress form as follows:

$$\partial(\sigma + K)/\partial r = -K[\Delta/(\pi/2)]2/H. \tag{8}$$

For the compression of an ideal plastic material (K = const) between plane anvils, subject to the boundary condition $\tau_K = K$, we have

$$\Delta = -\pi/2$$

and Eq. (8) in the form $\partial\sigma/\partial r = \partial p/\partial r = K2/H$ coincides with Eq. (1) of [3]. For the compression of an ideal plastic material between the profiled ends of the plungers, subject to the same boundary condition $\tau_K = K$, Eq. (8) becomes

$$\partial\sigma/\partial r = \partial p/\partial r = K[(\pi/2 - 2\beta)/\pi/2]2/H.$$

The final form of Eq. (8) is determined by the form of $K(\sigma)$ for the shear yield stress of the compressed material.

In our present analysis, the expression for the shear yield stress of the compressed material is derived from the concept of the generalized ideally plastic material, such as we take the compressed solid to be. The concept of an ideally plastic generalized solid was introduced in [6] after considering the envelopes of the Mohr circles for the greatest principal stresses. As envelopes of the Mohr circles we took straight lines inclined to the σ axis at an angle of $\rho = -\arctan \delta$ where δ is the internal-friction coefficient of the compressed

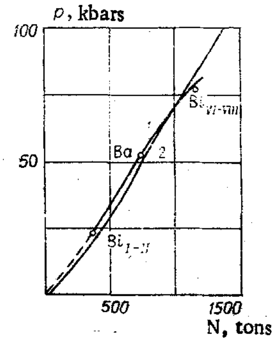

Fig. 1

material. From purely geometrical considerations the expression for the shear yield stress K assumes the following form:

$$K = K_0 \cos \rho + \sigma \sin \rho,$$

where K_0 is the shear yield stress at atmospheric pressure. The values of K_0 and ρ were determined from the data presented in [7, 8].

Equation (8) may now be solved subject to the boundary conditions generally accepted in the theory of the pressure treatment of metals, which are justified experimentally. On the free surfaces the normal and tangential stresses are taken as equal to zero. On the contact surfaces we distinguish three zones, each having its own characteristic laws governing the variation of the tangential frictional forces: a slip zone adjacent to the free surface of the solid, in which the contact frictional force is determined by the Coulomb law (the tangential stresses are proportional to the normal stresses); a zone of retardation, in which the contact frictional force is equal to the shear stress of the compressed material in its limiting state ($\tau_K = K \cos \rho$); a stagnation zone at the point of flow separation, in which the contact stress falls to zero. The size of the end (slip) zone in the absence of lubricant on the contact surfaces is of the order of the thickness of the compressed layer; only a slight error is created by neglecting this zone and considering that the zone of retardation begins immediately at the initial point of the contact surface. For the retardation zone Eq. (8) takes the form

$$\partial\sigma/\partial r = [(K_0 \cos \rho + \sigma \sin \rho)/(1 + \sin \rho)][(\pi/2 + \rho - 2\beta)/\pi/2]\ 2/H.$$

In the stagnation zone lying in the central part of the compressed solid we took the usual relationship for the tangential forces of friction $\tau_K = (r/r_0) K \cos \rho$, where r_0 is the coordinate corresponding to the beginning of the stagnation zone. Equation (8) then takes the form

$$\frac{\partial\sigma}{\partial r} = \frac{K_0 \cos \rho + \sigma \sin \rho}{1 + \sin \rho}\ \frac{\arcsin\left[\frac{r}{r_0} \sin\left(\frac{\pi}{2} + \rho\right)\right] - 2\beta}{\pi/2}\ \frac{2}{H}.$$

On the basis of experimental data regarding the compression of materials between plane slabs it was earlier [9] proposed that $r_0 = H$. In the case of profiled anvils it is difficult to establish the coordinates of the beginning of the stagnation zone on the basis of the recommendations made in [9]. In the present analysis we determined the beginning of the stagnation zone by comparing the displaced volume and the transmission capacity of the lateral surface for the value of r under consideration. The value of the displaced volume was $\pi r^2 v_z$ where v_z is the rate of relative approach of the plungers in the pressure device; the transmission capacity of the lateral surface for the same value of r is $2\pi r H v_r$, where v_r is the radial flow velocity averaged with respect to height. If the displaced volume exceeds the transmission capacity for the value of r under consideration (i.e., $\pi r^2 v_z > 2\pi r H v_r$), the compressed solid is embraced by the flow over its whole height and the section under consideration lies outside the stagnation zone. In the opposite situation ($\pi r^2 v_z < 2\pi r H v_r$) the compressed layer is not subject to the flow over its entire height, but only in the middle parts of the layer; this cross section accordingly lies within the stagnation zone. The coordinates of the boundary of the stagnation zone are determined by equating the displaced volume to the transmission capacity of the lateral surface for the value of r under consideration, i.e., $\pi r_0^2 v_z = 2\pi r_0 H v_r$.

For regions of the compressed layer close to its central section we may take $v_r \approx v_z$. Hence in order to determine the coordinates of the beginning of the stagnation zone it is sufficient to compare the area of the cross section perpendicular to the direction of mutual approach of the plungers with that of the lateral surface of the compressed layer for the same value of coordinate r. This comparison yields the following expression for the coordinate of the boundary of the stagnation zone:

$$r_0 = 2H(r_0).$$

From the same considerations the coordinates of the boundary of the stagnation zone in the case of plane flow are given by $x_0 = H(x_0)$; this agrees with the earlier recommendations [9] for the compression of material by plane anvils. After determining the average stress σ, all the components of the stress and pressure $p = \sigma + K/3$ in the median plane of the compressed solid may be determined.

From the pressure distribution in the median plane of the compressed layer we may now determine the required stress N and plot a calibration curve $N = f(p_0)$, where p_0 is the pressure in the center of the compressed layer.

Experimental and theoretical results relating to the calibration curve of a particular form of the pressure devices described in [4, 5], based on the equilibrium equation (4) expressed in stress form, are presented in Fig. 1; we see that the difference between the experimental and theoretical calibration curves (curves 1 and 2, respectively) is no greater than 7-8% for pressures up to 90 kbar (the tests extending to 100 kbar), and this may be regarded as satisfactory. The difference between calculation and experiment above 90 kbar may be due to some effect of plunger deformation not taken into account in the present analysis.

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